

10/3/17 "The will to exceed is important, but whats more important is the will to prepare"
-Bobby Knight

HW: "The Derivative and Tangent Line Problem HW" #1-5
Test 2 on Monday 10/16

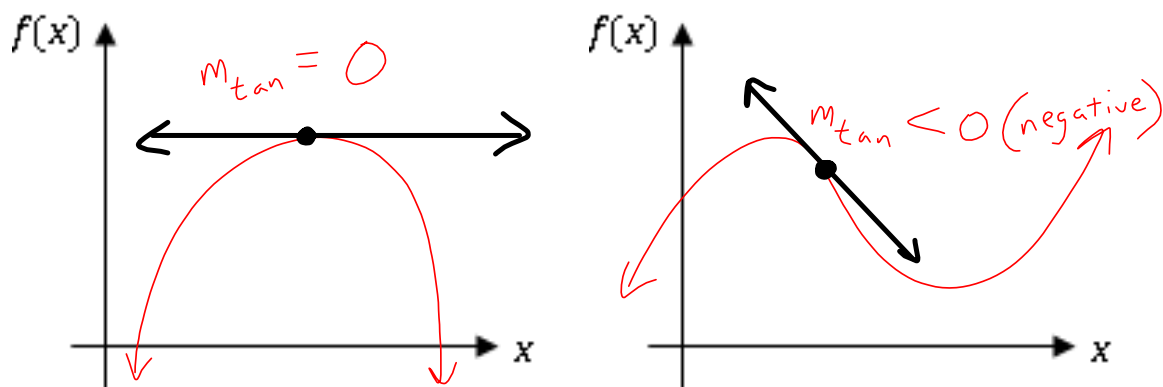
AIM: How do we find Derivatives?
Warm Up:

The Derivative and the Tangent Line Problem

Analytically: The definition of a derivative of f at x is the difference quotient (slope generator):

$$f'(x) = m_{tan} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Graphically: The derivative of a function at a point p is the slope of a tangent line to the graph of f at p .



Numerically: The derivative at a point is the limit of slopes of the secant lines or the limit of the difference quotient.

The derivative at a point $x = a$ is found by:

$$f'(a) = m_{tan} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$f(x) = x^2$
 $a = 2$

or by
$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \frac{x^2 - 4}{x - 2} = x + 2$$

 $2 + 2 = 4$

provided the limit exists.

For all a for which the limit exists,

$f'(x)$ is a function of a .

Pull

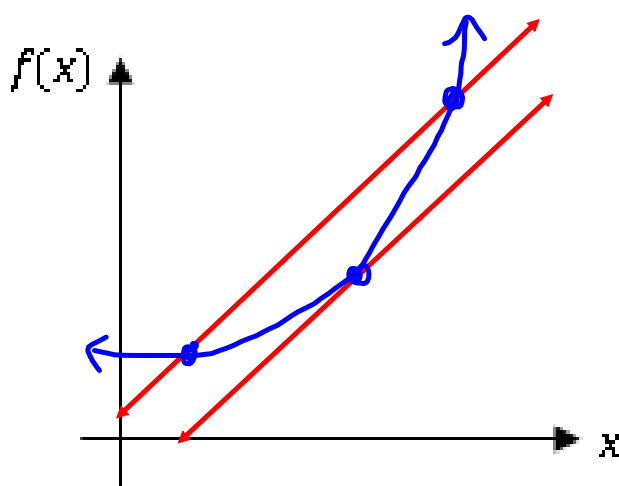
A function is differentiable at a given value of x if you can take the derivative of the function at that x value. In other words, $f(x)$ is differentiable at $x=c$ if $f'(c)$ exists. A function that is differentiable at every point of its domain has a derivative.

NOTATION FOR DERIVATIVES:

$f'(x)$	$\frac{dy}{dx}$	y'
$D_x[y]$	$\frac{d}{dx}[f(x)]$	$\frac{df}{dx}$

Using Limit Notation:

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$



EX #1: Use the definition of the derivative to find the slope of the graph $f(x)=3x-2$. Find $f'(-3)$.
Is the derivative a function?

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)-2 - 3x+2}{h}$$

$$\frac{\cancel{3x} + 3h - \cancel{2} - \cancel{3x} + \cancel{2}}{h}$$

$$\frac{3h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} 3$$

$$f'(x) = 3$$

$$\boxed{f'(-3) = 3}$$

yes,
derivative is
a function
(horizontal line)

EX #2: Find the slope of the tangent ^{derivative} lines to the graph of $f(x) = x^2 - 5$ at the points $(-2, -1)$ and $(1, -4)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 5 - x^2 + 5}{h}$$

$$\frac{\cancel{x^2} + 2xh + \cancel{h^2} - \cancel{5} - \cancel{x^2} + \cancel{5}}{h}$$

$$\frac{2xh + h^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} 2x + h$$

$$f'(x) = 2x$$

$$@ (-2, -1)$$

$$f'(-2) = 2(-2) = \boxed{-4}$$

$$@ (1, -4)$$

$$f'(1) = 2(1) = \boxed{2}$$

EX #3: For the function, $f(x) = x^3 - 2x$, find the derivative and evaluate the derivative at $x = 2, 0$, and -1 .

EX #4: Find $f'(x)$ for $f(x) = -\sqrt{x}$. Then find the slopes of the graph at the point $(4, -2)$.
 What is the behavior of the graph at $(0, 0)$?

derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{-\sqrt{x+h} - (-\sqrt{x})}{h}$$

$$= \frac{-\sqrt{x+h} + \sqrt{x}}{h} \cdot \frac{-\sqrt{x+h} - \sqrt{x}}{-\sqrt{x+h} - \sqrt{x}}$$

$$\frac{x+h-x}{h(-\sqrt{x+h}-\sqrt{x})} \Rightarrow \frac{h}{h(-\sqrt{x+h}-\sqrt{x})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{1}{-\sqrt{x+h}-\sqrt{x}}$$

$$f'(x) = \frac{1}{-2\sqrt{x}}$$

@ $(4, -2)$

$$f'(4) = \frac{1}{-2\sqrt{4}} = \frac{1}{-4}$$

@ $(0, 0)$

$$f'(0) = \frac{1}{0}$$

The derivative DNE
(end of graph)